

Azonosító  
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**ÉRETTSÉGI VIZSGA • 2022. október 18.**

# MATEMATIKA ANGOL NYELVEN

## EMELT SZINTŰ ÍRÁSBELI VIZSGA

minden vizsgázó számára

**2022. október 18. 8:00**

Időtartam: 300 perc

Pótlapok száma	
Tisztázati	
Piszkozati	

**OKTATÁSI HIVATAL**

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## Instructions to candidates

1. The time allowed for this examination paper is 300 minutes. When that time is up, you will have to stop working.
2. You may solve the problems in any order.
3. In part II, you are only required to solve four of the five problems. **When you have finished the examination, enter the number of the problem not selected in the square below.** *If it is not clear* for the examiner which problem you do not want to be assessed, the last problem in this examination paper will not be assessed.

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4. On solving the problems, you may use a calculator that cannot store and display textual information. You may also use any edition of the four-digit data tables. The use of any other electronic device or printed or written material is forbidden!
5. **Always write down the reasoning used to obtain the answers. A major part of the score will be awarded for this.**
6. **Make sure that calculations of intermediate results are also possible to follow.**
7. **The use of calculators** in the reasoning behind a particular solution **may be accepted without further mathematical explanation in case of the following operations:** addition, subtraction, multiplication, division, calculating powers and roots,  $n!$ ,  $\binom{n}{k}$ , replacing the tables found in the 4-digit Data Booklet (sin, cos, tan, log, and their inverse functions), approximate values of the numbers  $\pi$  and  $e$ , finding the solutions of the standard quadratic equation. No further explanation is needed when the calculator is used to find the mean and the standard deviation, as long as the text of the question does not explicitly require the candidate to show detailed work. **In any other cases, results obtained through the use of a calculator are considered as unexplained and points for such results will not be awarded.**
8. On solving the problems, theorems studied and given a name in class (e.g. the Pythagorean Theorem or the height theorem) do not need to be stated precisely. It is enough to refer to them by name, but their applicability needs to be briefly explained. Reference to other theorems will be fully accepted only if the theorem and all its conditions are stated correctly (proof is not required) and the applicability of the theorem to the given problem is explained.
9. Always state the final result (the answer to the question of the problem) in words, too!

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10. Write in pen. Diagrams may be drawn in pencil. The examiner is instructed not to mark anything written in pencil, other than diagrams. If you cancel any solution or part of a solution by crossing it over, it will not be assessed.
11. Only one solution to each problem will be assessed. In case of more than one attempt to solve a problem, **indicate clearly** which attempt you wish to be marked.
12. Please, **do not write in the grey rectangles**.

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## I.

1. The points  $C(-6; -2)$  and  $P(-3; 2)$  are given.

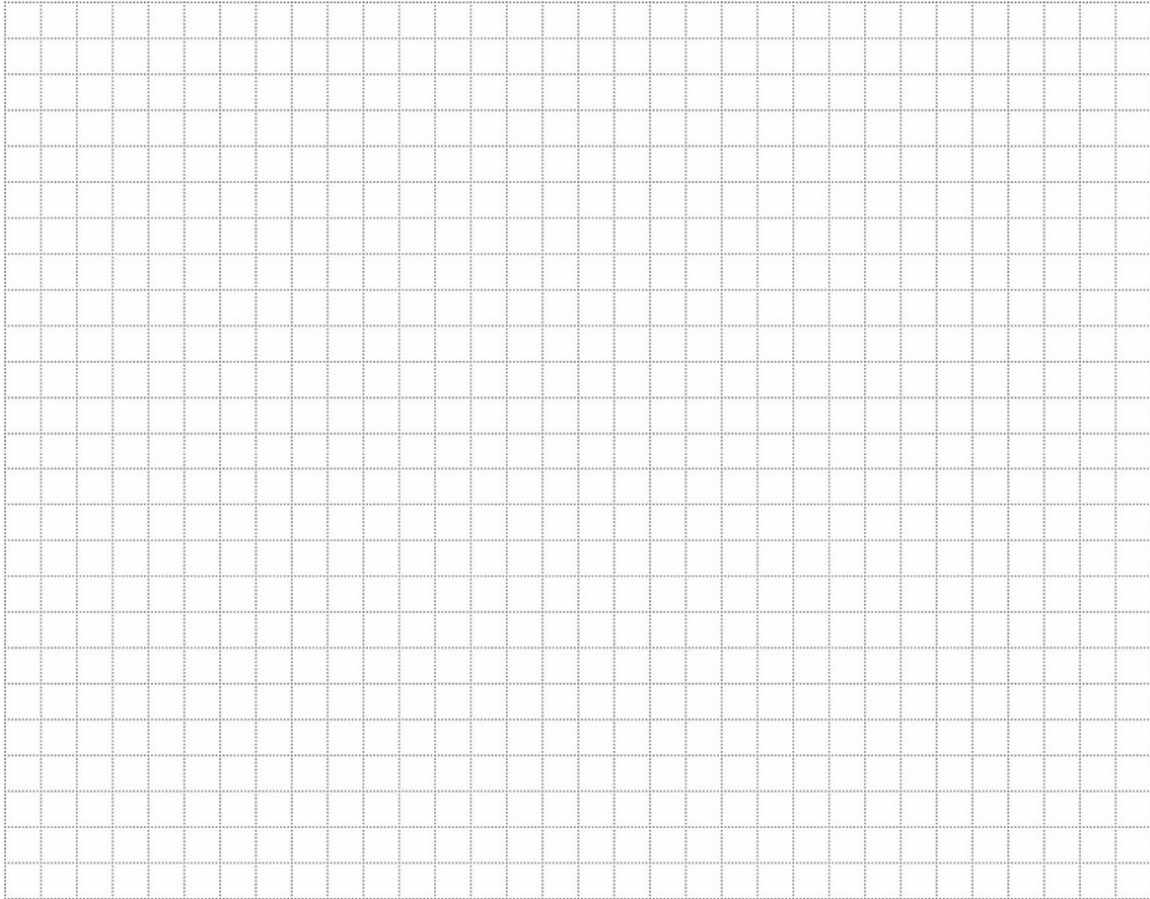
- a) The centre of the circle  $k$  is point  $C$  and it goes through point  $P$ . Give the equation of circle  $k$ .
- b) Give the equation of the tangent line drawn to circle  $k$  at point  $P$ .

A line is crossing through points  $C$  and  $P$ . This line and the two axes of the coordinate system form a right triangle.

- c) Determine the length of the radius of the circumcircle of this triangle.

<b>a)</b>	2 points	
<b>b)</b>	3 points	
<b>c)</b>	6 points	
<b>T.:</b>	11 points	

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2. Solve the following equations over the set of real numbers.

a)  $\sin^2 x = 3 \cos^2 x$

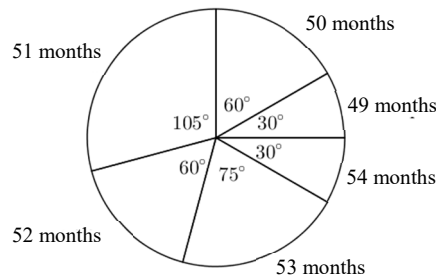
b)  $\log_3(x+8) + \log_3(x-2) - \log_3(x+4) = 1$

<b>a)</b>	6 points	
<b>b)</b>	8 points	
<b>T.:</b>	14 points	

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3. A company produces solar battery chargers. The lifespan of 24 of the company's products were recorded, the results are shown in the pie chart below.



- a) Fill in the table below and determine the mean and standard deviation of the lifespans of these 24 chargers.

lifespan (months)							
no. of chargers							

According to a more detailed study, 90% of the chargers manufactured by this company had a lifespan of at least 50 months (this can also be interpreted as a 0.9 probability for any randomly selected charger to have a lifespan of at least 50 months).

- b) Calculate the probability that out of 20 chargers delivered to a shop there will be no more than two chargers with a lifespan of less than 50 months.

It is also known that there is a probability of 0.75 that all of five randomly selected chargers will have a lifespan below 55 months.

- c) What is the probability that a single randomly selected charger will have a lifespan of at least 55 months?

a)	5 points	
b)	5 points	
c)	3 points	
<b>T.:</b>	13 points	



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4. Consider the functions  $f(x) = \sin x$  and  $g(x) = \left(\frac{2x}{\pi}\right)^2$ , ( $x \in \mathbf{R}$ ).

- a) Prove that the graphs of both of the functions given above cross through the origin and the point  $\left(\frac{\pi}{2}; 1\right)$ .
- b) Determine the area of the figure between the graphs of the two functions where  $x \in \left[0; \frac{\pi}{2}\right]$ !

Consider the sequence  $a_n = \frac{2 + 2\pi n}{n}$ , ( $n \in \mathbf{N}^+$ ).

- c) Prove that this sequence is (strictly) monotone decreasing, that it is bounded, and also determine its limit.

<b>a)</b>	3 points	
<b>b)</b>	5 points	
<b>c)</b>	5 points	
<b>T.:</b>	13 points	

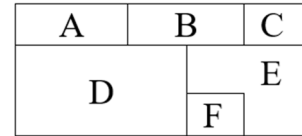
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## II.

**You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.**

5. A rectangle has been divided into six regions, as shown in the diagram. The regions A, B, C, D, E and F must be coloured, such that regions of the same colour must not be adjacent to one another. The colours red, blue, green and yellow may be used. (Every region must be coloured in one of the four colours but it is not required that all four colours are used.)



- a) How many different ways are there to colour the rectangle if the regions A and C must be of **different** colours?

$A$ ,  $B$ ,  $C$ ,  $D$ ,  $E$  and  $F$  are non-negative numbers. It is known that:

- (1)  $A = 6$  and  $D = 8$ ;
- (2)  $B$  is the arithmetic mean of  $A$  and  $C$ ;
- (3)  $F$  is the geometric mean of  $D$  and  $E$ ;
- (4)  $F$  is 1 greater than  $B$ ;
- (5)  $E$  is 2 greater than  $C$ .

- b) Determine the value of the unknown numbers.

a)	7 points	
b)	9 points	
T.:	16 points	

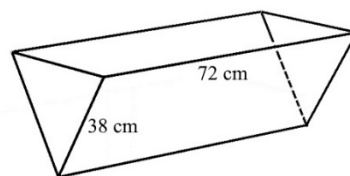
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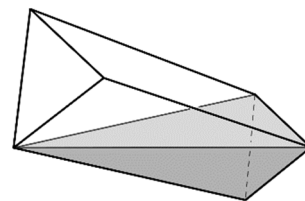
**You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.**

6. A horizontal plot of land, used for pasture, is up for sale. The shape of the plot is a convex quadrilateral, let the vertices be  $A$ ,  $B$ ,  $C$ , and  $D$ . Three sides of the plot are  $AB = 126$  m,  $BC = 65$  m,  $CD = 80$  m, two angles are  $\angle ABC = 122.5^\circ$  and  $\angle ADC = 90^\circ$ . The land is advertised to be 0.9 hectare.
- a) By what percentage is the real area of the land greater than the advertised area? (1 ha = 10 000 m<sup>2</sup>)

A trough is used for watering animals. The shape of the trough is a triangular right prism. In its horizontal position, the trough is open on the top, this face of the prism is parallel to the horizontal ground, while the triangular faces are perpendicular to the level ground (see diagram). All sides of the regular triangles making two faces of the prism are 38 cm, the two rectangular faces are 38 cm × 72 cm.



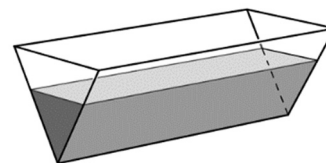
At the beginning, the trough is horizontal, and is full of water. One end of the trough is then lifted, and so some of the water pours out. The water level is now at the bottom vertex of the triangle on the left and at the top side of the triangle on the right, as shown in the diagram.



- b) Prove that there are 15 litres of water in the trough now. (Rounded to the nearest integer.)

The trough is then lowered to its original position.

- c) How many cm deep is the water in the trough now?



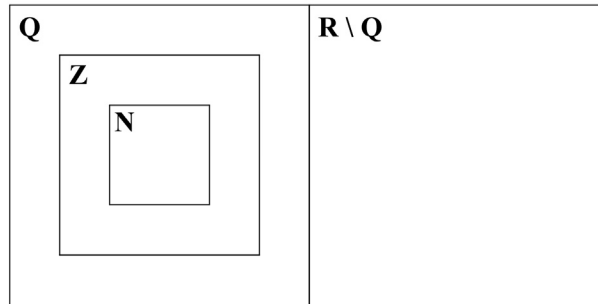
a)	6 points	
b)	5 points	
c)	5 points	
T.:	16 points	

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**You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.**

7. a) Consider the function  $f(x) = 3^{-x}$  ( $x \in \mathbf{R}$ ). Place the values  $f(-2)$ ,  $f(0.5)$  and  $f(5)$  into the appropriate regions of the set diagram below.

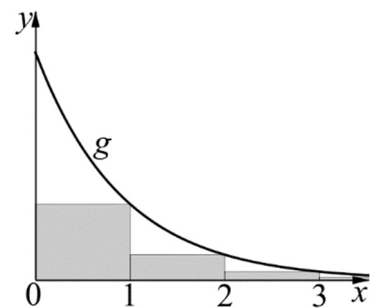


The numbers  $3^{-2}$ ,  $3^{-7}$ ,  $3^{-12}$ ,  $1-\sqrt{2}$  and  $\frac{1}{\sqrt{2}-1}$  are written to vertices  $A, B, C, D, E$  of a 5-point simple graph, respectively. Any two vertices are connected with an edge if and only if the **sum** of the two numbers at the ends of the edge is rational.

- b) How many edges does this 5-point graph have?

Rectangles are written into the region of the coordinate system bounded by the axes and the graph of the function  $g(x) = 3^{-x}$  ( $x \geq 0$ ). The rectangles are adjacent to each other, one side is 1 unit on the  $x$  axis, and one vertex is on the graph of function  $g$ .

One vertex of the first rectangle is the origin, the opposite vertex is the point  $(1; g(1))$ . Vertices of successive rectangles are  $(2; g(2))$ ,  $(3; g(3))$ , etc. as shown in the diagram (not drawn to scale!).



Let  $n$  be the greatest positive integer for which  $g(n) - g(n+1) > 10^{-6}$  is true.

- c) Calculate the sum of the areas of the first  $n$  rectangles.

a)	3 points	
b)	5 points	
c)	8 points	
<b>T.:</b>	16 points	



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**You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.**

- 8.** One edge of a cuboid is 4 dm, another is 2 dm long. The volume of the cuboid is  $72 \text{ dm}^3$ .

a) Calculate the total surface area of the cuboid.

The volume of a cuboid is  $72 \text{ dm}^3$ . One edge of the cuboid is twice as long as another edge.

- b) Of all such cuboids, give the lengths of the edges of the one that has minimal surface area.
- c) How many different possible ways are there to select three of the vertices of the cuboid, such that the plane across these three vertices does not contain any other vertex of the cuboid?

a)	2 points	
b)	8 points	
c)	6 points	
<b>T.:</b>	16 points	

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**You are required to solve any four out of the problems 5 to 9. Write the number of the problem NOT selected in the blank square on page 2.**

9. Raffle tickets are sold at a charity event. 5 blue tickets and 3 green tickets cost 6700 Ft, 3 blue and 2 green tickets cost 4200 Ft.

a) What does a single blue ticket cost and what is the price of a single green ticket?

40% of the tickets is blue, the other 60% are green. The table below shows the proportions of the various prizes the different tickets may win (e.g. 35% of **all blue** tickets win a prize item).

	blue	green
wins a prize item	35%	40%
wins a 1000 Ft book voucher	20%	30%
wins a 5000 Ft book voucher	5%	–
does not win	40%	30%

A single raffle ticket is randomly selected. Let  $A$  be the event that this ticket wins a prize item, and let  $B$  be the event that this ticket is blue.

- b) Prove that  $P(A) = 0.38$ .  
Calculate the conditional probability  $P(B | A)$ .  
Are events  $A$  and  $B$  independent?
- c) Calculate the expected value of a single blue ticket, prize items being valued at 500 Ft each.

a)	5 points	
b)	8 points	
c)	3 points	
<b>T.:</b>	16 points	

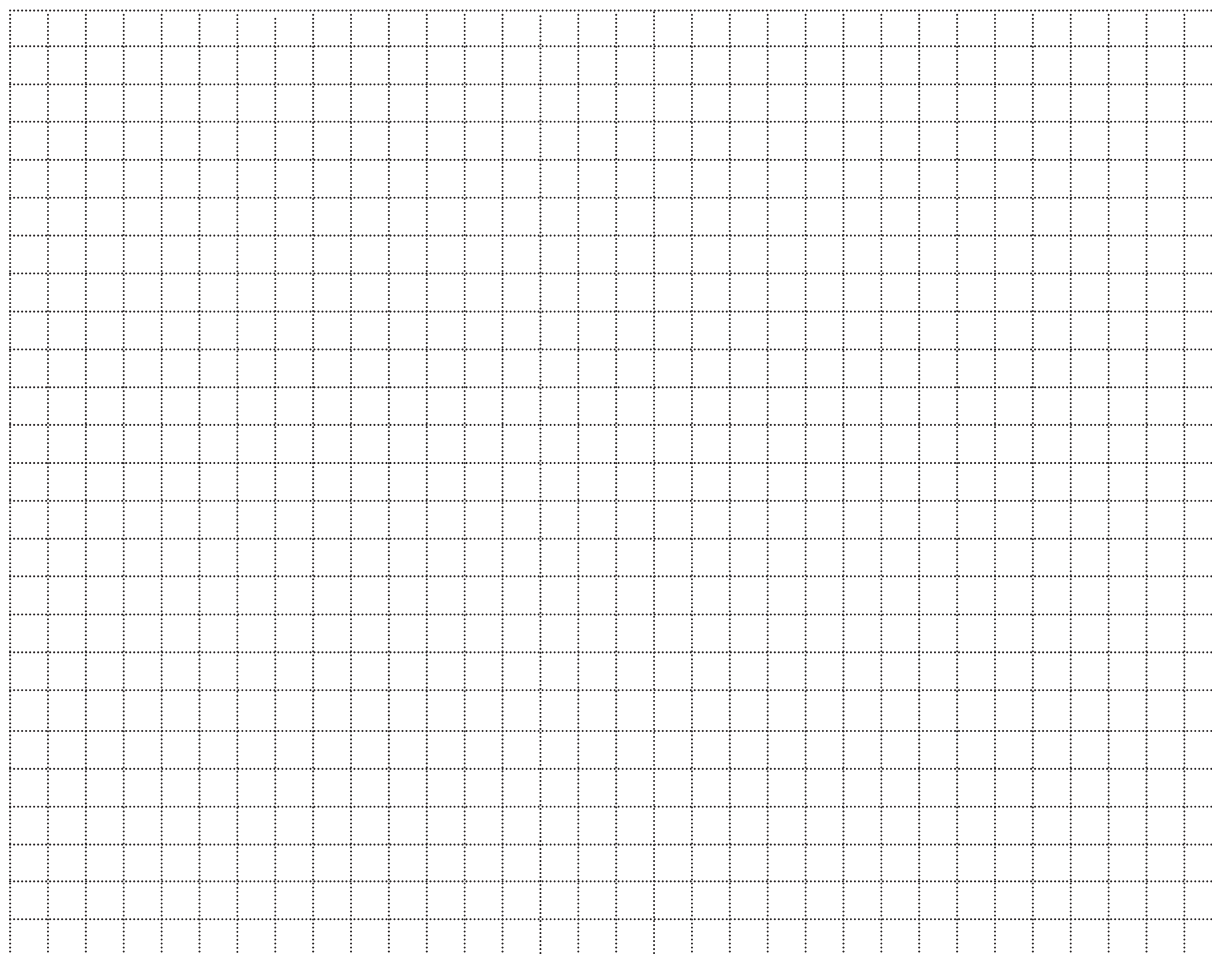
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	Number of problem	score			
		maximum	awarded	maximum	awarded
Part I	1.	11		<b>51</b>	
	2.	14			
	3.	13			
	4.	13			
Part II		16		<b>64</b>	
		16			
		16			
		16			
		← problem not selected			
<b>Total score on written examination</b>				<b>115</b>	

\_\_\_\_\_ date

\_\_\_\_\_ examiner

	pontszáma <b>egész</b> <b>számra</b> kerekítve	
	elért	programba beírt
I. rész		
II. rész		

\_\_\_\_\_ dátum

\_\_\_\_\_ dátum

\_\_\_\_\_ javító tanár

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